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Structural and Vibrational Bedrock Properties in Sweden

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Bedrock properties, of great concern in seismology as well as in numerous practical applications, are elucidated by means of short-period seismic wave propagation. Field work in central Sweden revealed the existence of a superficial granitic layer 1.4 km thick with lower wave velocities than the layer below. The brittle nature of the top layer is evidenced by low quality factors and is related to rockbursts in the vicinity of mines. By spectral techniques, the permanent Swedish network is used for quantitative estimation of ground motion (acceleration, regional magnitude, source energy) due to regional seismic events. For localization of teleseismic events and other signal combination, the whole network can be treated as a continental array, because of excellent signal coherence between the stations. Variations of sensitivity among the stations depend mainly upon relation to microseismic sources around the coasts. Our aim has been to elucidate the importance of bedrock properties for a number of different problems in pure and applied seismology.

1. INTRODUCTION

From a geological point of view, the landscape of Sweden generally exhibits a combination of the oldest and the newest formations. The bedrock, usually of granite or gneiss of Precambrian age, is covered by varying thicknesses of postglacial Quaternary deposits, such as clay, gravel, and sand. Formations of intermediate age, e.g., sedimentary rocks, as sandstone, limestone, etc., occur in more isolated parts of the country. With a bedrock outcropping at numerous places and being close to the free surface almost everywhere, it is natural that its properties are of great concern to all kinds of major constructions, such as of dams and nuclear power plants, as well as for the storage of radioactive waste material. Similarly, its properties are of paramount significance in the search for sensitive locations for seismograph stations.

In the present paper, I summarize results of bedrock investigations made during the last decade at the Seismological Institute of Uppsala, with main emphasis on still unpublished material. The results are all based on measurements of seismic wave propagation, almost exclusively of short-period waves, either in field works or from the six permanent stations of the Swedish network. While the results may be valid for other areas of similar structure, especially shield areas, the methods involved are of more general applicability.

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2. Notation

A	ground acceleration amplitude	microns/sec^2
$A(t)$	ground acceleration versus time graph	microns/sec^2
cps	cycles per second	
d	trace displacement amplitude	mm
D	ground displacement amplitude	microns
E	total seismic wave energy	ergs
F	Fourier spectrum	
$F(\Delta, T)$	calibrating term in magnitude formula	
h	depth	km
I	seismograph response	mm/microns
log	decadic logarithm	
m	body-wave magnitude	
M	surface-wave magnitude	
M_L	regional magnitude	
n	distance exponent in attenuation formulas	
Q	quality factor	
RS	response spectrum	
t	time	sec
T	wave period	sec
V	wave velocity	km/sec
WA	standard Wood-Anderson seismograph	
y	relative mass displacement	microns
Δ	epicentral distance	km
Δ'	hypocentral distance $= (\Delta^2 + h^2)^{1/2}$	km
η	damping ratio	
μ	microns $= 10^{-3}$ mm	
$\sigma(\Delta)$	dispersion correction in formula for M_L	
τ	travel time	sec
$\phi(\omega)$	seismograph phase response	radians
ω	angular frequency	radians/sec
ω_0	natural angular frequency of a system	radians/sec

3. Structural Properties

3.1. *Superficial Granitic Layering*

On the basis of observed dispersion of short-period Rayleigh waves (Rg), a superficial granitic layer of about 1 to 2 km in thickness has been inferred [6]. The deep crustal structure of Fennoscandia has been investigated by several institutes, using numerous intersecting profiles over the area. For example, by combination of a number of profiles over Sweden, an average crustal structure could be deduced, primarily for use in localization of regional earthquakes [3]. The layering can be considered as a typical continental shield structure. However, these profiles are inefficient in elucidating superficial layering.

With the purpose of establishing superficial granitic layering, a seismic refraction study was undertaken in central Sweden in the Grängesberg mining area (60.1° N, 15.0° E) in 1975 [9]. A linear profile 17.5 km in length extending in a northwesterly direction from the mines was studied by a dense coverage of geophones, the total number of measuring points being 73. Regular mining blasts at Grängesberg were used as seismic wave sources.

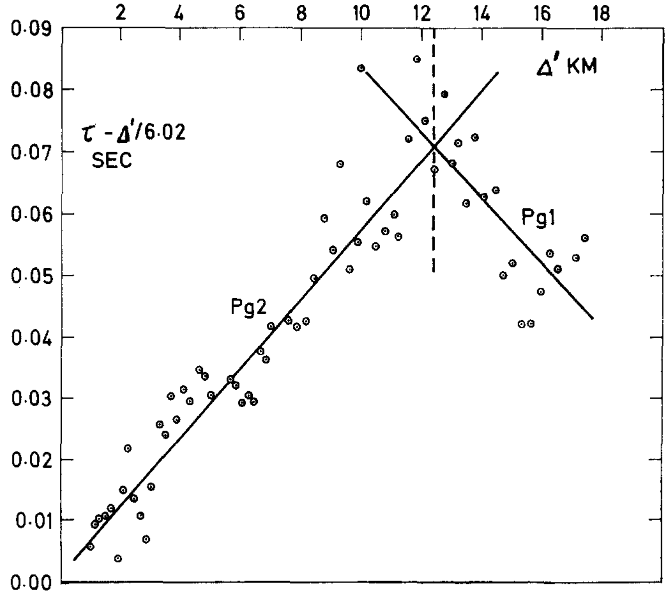

FIG. 1. Reduced travel time graph with least squares solutions for Pg2 and Pg1.

The travel time graphs of P and S suggest two different phases for each wave type, clearest for P, with a crossover near 12-km distance (Fig. 1). First waves recorded at shorter distances are interpreted as direct waves, denoted Pg2 and Sg2, in a superficial layer, and those recorded at larger distances are refracted waves, denoted Pg1 and Sg1. The following least squares solutions are obtained:

$$\begin{aligned}\tau(\mathrm{Pg2}) &= (0.001 \pm 0.002) + \Delta'/(5.82 \pm 0.01),\\ \tau(\mathrm{Pg1}) &= (0.138 \pm 0.016) + \Delta/(6.22 \pm 0.04),\\ \tau(\mathrm{Sg2}) &= (0.031 \pm 0.017) + \Delta'/(3.38 \pm 0.03),\\ \tau(\mathrm{Sg1}) &= (0.286 \pm 0.096) + \Delta/(3.62 \pm 010).\end{aligned} \tag{1}$$

The wave notation is taken over from [3], where from seismic explosion work in

Scandinavia, clear evidence was found for two Pg waves and two Sg waves. In [3], no explanation is presented, but the new findings here may contribute to an explanation, even though our present sampling of crustal information is rather shallow. According to Eq. (1) the two P velocities are significantly different from each other as are the two S velocities. This suggests that the boundary is relatively sharp, or at least, if a transition zone is admitted, this has to be rather narrow.

Our recordings have thus strongly confirmed the existence of a thin superficial layer. Using Eq. (1), its thickness is calculated to be 1.4 $\pm$ 0.2 km from P wave data and 1.6 $\pm$ 0.9 km from the less reliable S wave data. Therefore, the adopted average thickness is 1.4 km. The assumption of horizontal layering proves to be very reasonable.

Proceeding to a finer interpretation, there is some indication that the Pg2 and Sg2 velocities are not exactly constant, but increase slightly with depth, for Pg2 according to the following equation:

$$V(\mathrm{Pg2}) = 5.79 + 0.043\, h \qquad 0 \leqslant h \leqslant 1.4 \text{ km}. \tag{2}$$

The velocity gradient (0.043) shows good agreement with results from laboratory investigations of the influence of pressure and temperature on wave velocities. The effect is essentially due to downward increasing pressure with ensuing compaction, closing of cracks, etc. On the other hand, the discontinuity at 1.4-km depth is sharp and must involve other, sudden changes, e.g., of composition. Deep drillings to this depth are not yet available in this area to establish the exact nature of the 1.4-km discontinuity.

3.2. *Source and Path Properties of the Superficial Layer*

The existence of a distinct superficial granitic layer entails a number of phenomena, regarding both seismic events and wave propagation. Even though the discontinuity at 1.4-km depth remains unexplained, the top layer certainly exhibits much more of a brittle structure than the competent granitic rock below the discontinuity. The term "brittle" is here used in a general sense, indicating an ability to break rather than to yield when exposed to stress. Therefore such a layer is characterized by fracturing, as distinct from the more competent rock below. This is related to the existence of rockbursts in Sweden, especially around iron ore mines. The excavation in the mines leads to redistribution of the natural stress field with the consequence of brittle failure at certain points. A relatively large rockburst at Grängesberg on August 30, 1974, has been followed by an aftershock sequence of nearly 500 events in 3 yr. This series is the first aftershock sequence in Sweden that has permitted a detailed study. Its characteristics are very similar to those of aftershock sequences of major earthquakes. This series at Grängesberg stimulated a special research project at our institute, which has been reviewed in [7].

Another expression of the brittle nature of the upper layer is its quality factor Q which is lower than for the layer below because of wave scattering. This is evident

TABLE I

Selected Quality Factors for Granitic Layers

Wave	Period sec	Quality factor Q	Responsible layer	Reference
Pg2	0.02	50–80	Upper granite	[9]
Pg1	0.02	120	1.4 km discontinuity	[9]
Rg	0.75	180–300	Upper granite	[6]
Sg1	0.4–0.7	1060	Lower granite	[8]
Sg1	0.7–2.0	1830	Lower granite	[8]

from a summary of our observations (Table I), which illustrates effects both due to different layers and different wave periods. For all investigated waves, the periods cover a rather narrow range as indicated. Following up the amplitude variation with distance for any given period permits then calculation of Q. In Ref. [9] use is made of the seismic refraction profile, mentioned in Section 3.1, in [6] readings are made from rockburst records of our permanent network and in [8] the results are based on spectral analysis of our records of regional events, further described in Section 4.1 below.

That strong wave scattering exists in the superficial layer is also evidenced by the rapid decay of surface-generated seismic noise with depth [2, 4]. A number of geophone measurements of seismic noise at various levels down to about 750-m depth in three mines in central Sweden can be summarized in the following relation:

$$D \sim e^{-2.6\Delta'}/\Delta' \qquad 0.03 \text{ sec} \leqslant T \leqslant 0.13 \text{ sec}. \tag{3}$$

Equation (3) represents the observations well, except for the uppermost 100 m where the decrease with depth is still more rapid. As an average it is found that the noise amplitude decreases to 25 % of its surface value at a depth of 50 m, to 13 % at 100 m, to 6 % at 200 m depth, and is less than 1 % of the surface value at depths exceeding 500 m. The corresponding quality factors are remarkably low, clearly indicative of surface fracturing of the granite.

The observed dispersion of Rg waves [6] from near-surface events (explosions, rockbursts, not deeper than around 2 km) is well explained by the top granitic layer. However, quantitative comparisons demonstrate that for full agreement with observations it is necessary to include some tens of meters of low-velocity Quaternary deposits [9].

Layered media are expected to lead to reverberation of incoming seismic waves, which may disturb the records, especially for stations on sedimentary layers. However, thicknesses of layers have to be equal to or larger than the wavelengths of incoming waves for significant reverberation. For short-period teleseismic P waves (period around 1 sec), the wavelengths are about four times the thickness of the superficial granitic layer. For our stations, all located on bedrock and lacking other superficial layering, practically only the total crust produces discernable reverberation for P waves. Conversely, the observed reverberation can be used to estimate the total crustal thickness. A series of such investigations using spectral methods and with special reference to Swedish conditions are summarized in [19]. It is different with short-period Sgl waves from regional events. With periods of only 0.4 to 0.5 sec, the wavelengths are then comparable to the thickness of the superficial granite. This means that Sgl reverberation could show up in our short-period records of regional events, contributing to the complexity of the Sgl wave train.

We have anticipated a brittle structure of the upper granite. Nevertheless, the superficial appearance of the Swedish granite is quite different from what the present author has experienced, for example, from granitic outcrops in southern California. Compared to these, the superficial granite in Sweden appears competent and of good quality.

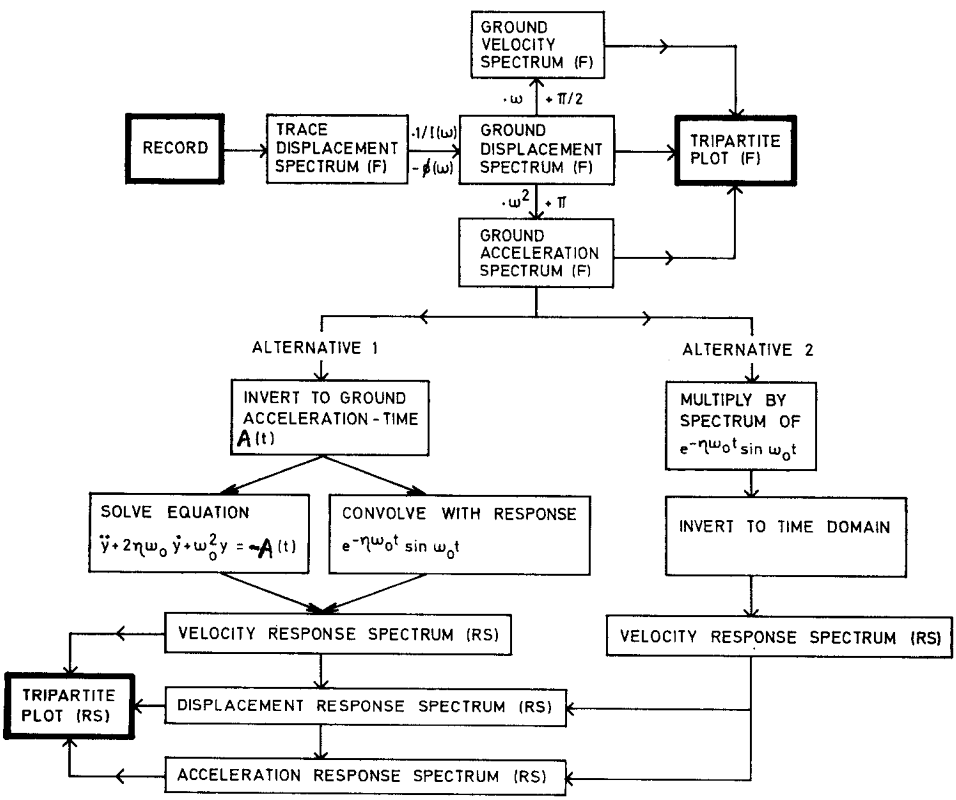

FIG. 2. Flow chart for the operations leading to tripartite diagrams of Fourier spectra and of response spectra.

4. Vibrational Properties

4.1. *Regional Seismic Events*

In order to fully evaluate ground vibrations from regional and local seismic events we should be able to record fairly high frequencies, up to 100 cps and over. This generally requires special installations of accelerometers, which are not available except in special areas. Moreover, it would be necessary to have such installations operating during a longer period of time in order to accumulate enough observational material. This is particularly true in an area like Sweden with a comparatively low seismicity level. In order to circumvent these obstacles, at least partially, we set out to examine our regular permanent records by the same spectral methods as ordinarily applied to accelerometer records. This approach has the advantage that enough observational material is immediately at our disposal. On the other hand, a certain drawback is the limitation towards higher frequencies, as the permanent short-period records allow us to depict spectra up to 10 cps at most, and they become less reliable already for frequencies above 2.5 cps. Nevertheless, we found this attack on the problem worthwhile.

In deducing Fourier spectra and response spectra, we could thus follow available

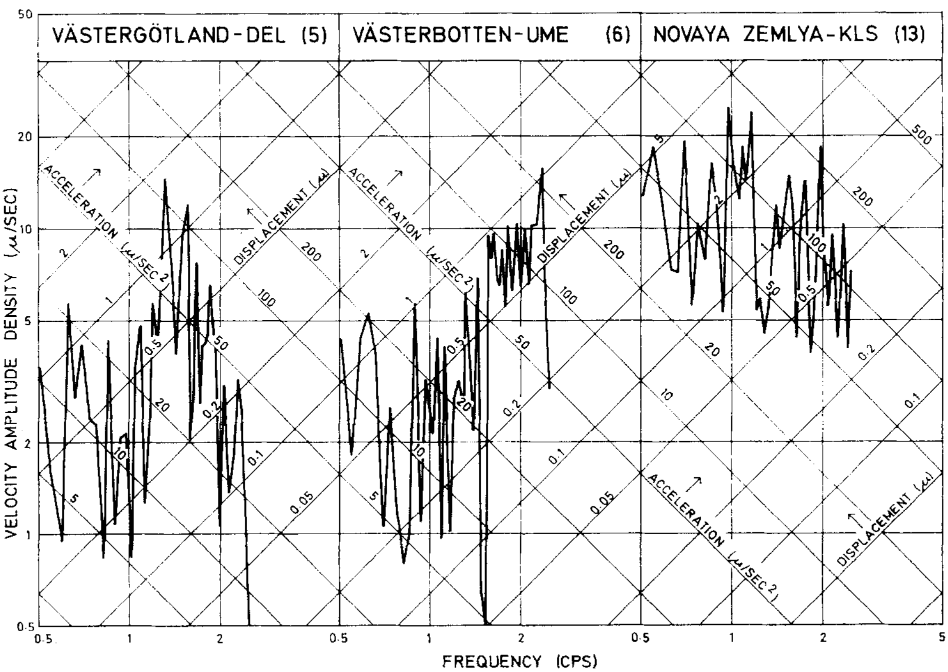

Fig. 3. Fourier tripartite spectra of three typical cases (Table II). In all three cases, the signal-to-noise ratio is very high, and the noise does not contribute more than a few percent to the depicted spectra.

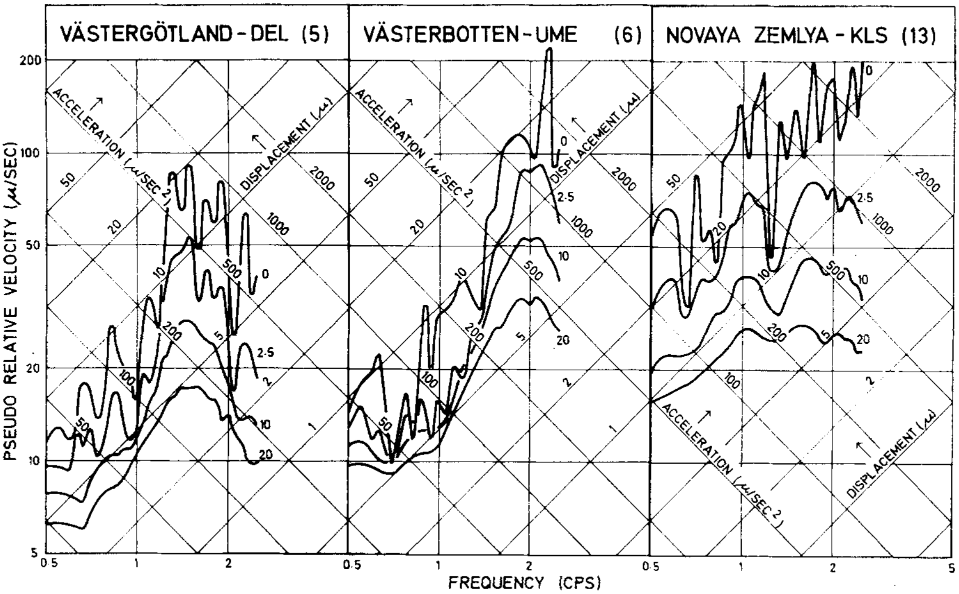

FIG. 4. Response tripartite spectra of three typical cases (Table II). Each graph contains four curves, corresponding to damping ratios of 0, 2.5, 10, and 20% of critical damping.

TABLE II

Events and Stations Used in Figs. 3 and 4.

(a) Events

Number	Region	Date	Location	Magnitude m
5	Västergötland (Sweden)	Apr. 11, 1973	58.8°N, 13.4°E	4.7
6	Västerbotten (Sweden)	Sep. 28, 1962	64.5°N, 20.7°E	4.7
13	Novaya Zemlya (USSR)	Oct. 21, 1967	73.5°N, 54.5°E	6.3

(b) Stations

Name	Location	Short-period seismograph	Wave
Delary (DEL)	56°28.2′N, 13°52.2′E	Grenet-Coulomb Z	Sg1
Umeå (UME)	63°48.9′N, 20°14.2′E	Benioff Z (WWSSN)	Sg1
Karlskrona (KLS)	56°09.9′N, 15°35.5′E	Grenet-Coulomb Z	P

methods (see, for example, [5]). Figure 2 gives a flow chart for the computation and Figs. 3 and 4 show some examples of computed spectra, with explanations in Table II. The spectra represent the maximum motion in each record, i.e., Sg1 for the two regional earthquakes (Västergötland and Västerbotten, Sweden) and P for the underground nuclear explosion (Novaya Zemlya). The computations shown in Fig. 2 are self-explanatory and do not deviate from those customary in seismology nowadays. There is only one point that needs to be especially emphasized. The acceleration-time graph $A(t)$ is not directly available as the case is when accelerograph records exist. On the contrary, in the present case the graph $A(t)$ has to be calculated by passing over the Fourier spectrum and inverting this back to the time domain. However, this inconvenience is more than offset by the advantages of having large amounts of records available from the standard networks just referred to. For more details, see [8].

However, beyond such spot information, it appears desirable to use the measurements for deducing ground motion under more general conditions. The path to this goal has to pass over an estimation of the wave attenuation. By combining 26 different paths crossing each other over the Swedish territory to our permanent stations, reliable attenuation estimates of Sg1 waves could be made. The spectra permit us to get the attenuation for each period separately, together covering the period range recorded by short-period seismographs (Table I). Detailed results are reported in [8].

Having the attenuation for Sweden established, it is a short step to develop the regional magnitude scale M_L. Its calculation is obvious from the flow chart in Fig. 5, corresponding to the following formula:

$$\begin{aligned} M_L &= \log D + n(\omega)\log(\Delta/100) + \log I_{\mathrm{WA}}(\omega) + \sigma(\Delta) \\ &= \log(100D) + F(\Delta, T), \end{aligned} \tag{4}$$

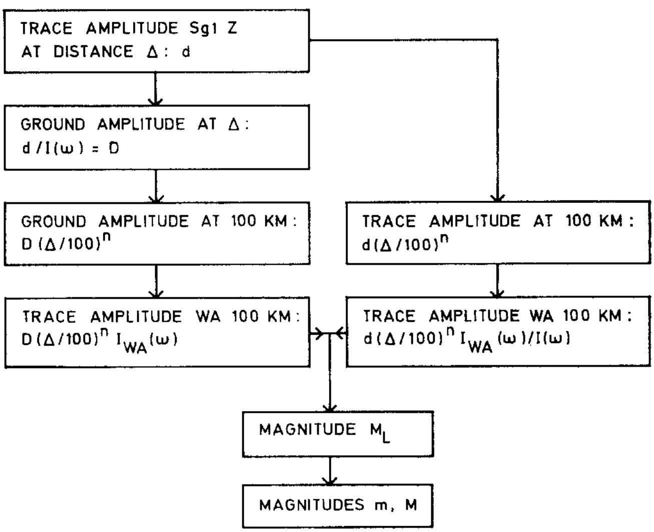

FIG. 5. Flow chart for computation of magnitudes M_L, m, and M.

where the factor 100 of D is included for computational convenience. In calculating M_L there are essentially two steps involved: (1) reduction of the observed amplitude to an epicentral distance of 100 km; (2) conversion of the result of (1) into what a standard Wood–Anderson seismograph would record. While step (2) is a purely instrumental problem, which can be solved accurately as soon as reliable response curves are at hand, step (1) depends on the regional attenuation. In too many studies, lack of information of the regional attenuation has prompted the use of attenuation values taken over from some other region, e.g., California, which may be far from applicable in other regions.

To complete the calculation of ground motion due to regional Swedish events, we deduced relations for the seismic wave energy E (ergs) and for the acceleration A (microns/sec^2) as well as between M_L and the usual body- and surface-wave magnitudes m and M. The results can be summarized in the following formulas, for whose deduction I refer to [8]:

$$\begin{aligned} \log E &= (12.30 \pm 0.35) + (1.27 \pm 0.12)\, M_L\,, \\ \log A &= M_L - 0.40 - 2 \log T - F(\Delta, T), \\ m &= 2.93 + 0.49 M_L\,, \\ M &= 0.04 + 0.88 M_L\,. \end{aligned} \tag{5}$$

For any recorded event occurring in Sweden or neighboring areas, Eqs. (4) and (5) permit an estimation of the ground motion experienced at any point within an epicentral distance of up to about 1,500 km. The motion thus obtained is characteristic for the bedrock, as our stations are located on this.

4.2. *Teleseismic Events*

Even a superficial examination of our network records of teleseismic P waves reveals great similarity between the stations. This impression is confirmed by detailed investigation, including calculation of coherence and cross-correlation between signals at different stations of our network. The strong similarity in the time domain implies a corresponding strong similarity in the frequency domain. These results, documented in [14, 15], were later extended to long-period S waves [21] (see also [4].) The main reason for the great signal similarity between our stations is probably the absence of sediments. Sedimentary layers with ensuing reverberation tend to complicate records, which is not the case with our stations.

This property gave the first stimulus to our launching of the idea of the continental array, originally termed SLASA = Super-Large Aperture Seismic Array, a concept and a method which later have been successfully applied in other areas, especially in North America. The great similarity among the signals suggested the application of array techniques, especially combination of station signals with the aim of improving the signal-to-noise ratio and the accuracy of readings.

The combination of Swedish and Finnish stations, all on excellent bedrock, proves advantageous for the reason that it makes the total network more nearly circular, whereas the Swedish alone is more elongated. Thus, array techniques were adopted in studying the ensemble of Swedish and Finnish station records both for the deduction of mantle wave velocity distribution [13] and for the localization of teleseisms [20]. The latter application proves to yield results with an accuracy comparable to or even better than that obtained from large arrays. Superior in sensitivity but hardly in localization accuracy, some of these with hundreds of sensors are handicapped by covering only a small area and especially by complicated underground structures. A 1-yr excellent operation of a triangular array of about 100 km side in the Uppsala area demonstrates the great significance of selecting the best possible bedrock for successful operation of arrays [10, 11, 12, 16, 18].

In spite of the great signal similarity among the stations, all located on bedrock of similar structure, there are still differences in their sensitivity to teleseismic P waves. Expressing the sensitivity as the number of recorded P waves, our station sensitivity exhibits clear geographical distribution with its maximum in the interior part of central Sweden (in the vicinity of our station Uddeholm, 60°05.4′ N, 13°36.4′ E). The mapping of sensitivity was based on our permanent network supplemented by recordings at about a dozen different places all over the country, each operating for a few months [1, 4]. The regular sensitivity distribution could be due to some variation in crustal properties, including focusing and defocusing effects, but primarily it is due to location in relation to noise (microseism) sources. As such the surrounding oceans dominate, as evidenced by a special study [17]. The distribution of microseisms can be explained as originating from quasi-linear sources along active coast lines and spreading out in all directions.

Even though the sensitivity of our seismograph stations ranks comparatively high by international standards, still more sensitive seismograph recordings in Sweden could be expected for the Uddeholm area at a depth below 1.4 km, using borehole seismometers. Considering the good quality of the Swedish rock, especially at this depth, such recordings would probably surpass everything so far known from single seismometers. In fact, in 1967 the present author worked out plans for such an installation at 2 km depth in the Uddeholm area. Cost estimates and a plan for the work were acquired from a specialist firm in Stockholm. It was estimated that one full year of continuous drilling was needed to reach the depth of 2 km in hard granite. Unfortunately, lack of funds prevented the realization of this project.

A related practical problem of great current importance concerns the storage of radioactive waste material. From our measurements it results that a storage below the 1.4 km discontinuity would be far more recommendable than a storage in the superficial layer, also that areas of mining activity or other rock excavation work should be avoided.

5. Conclusions

There are far more seismograph stations around the globe than there are seismic arrays or continuous accelerometer recordings. Together with a fairly good geographical coverage of networks, this fact should be used to advantage. In this paper I hope to have demonstrated how regular networks can be used as substitutes for accelerometer records in order to deduce the ground motion from local and regional seismic events, as long as very high frequencies can be disregarded. Likewise, I have pointed to the advantage which can be made of regular seismic networks by applying array processing methods, as soon as the stations exhibit enough signal similarity. These are both methodical viewpoints of fairly general validity.

Besides, I hope to have emphasized the significance of detailed knowledge of bedrock properties for fruitful seismic studies and interpretations. The bedrock under the stations constitutes an essential factor in the shaping of the records. In addition to the purely seismological applications, an accurate knowledge of the bedrock is nowadays mandatory for many constructional purposes. It is not only in seismic countries that due regard has to be taken to seismic phenomena in planning and constructing nuclear power plants and for the storage of radioactive waste material. Even in such a relatively quiet country as Sweden, seismically generated ground motion is nowadays considered as a significant element, incorporated in the planning stage. Moreover, mining companies have their attention directed to rockbursts around the mines.

Acknowledgments

I like to acknowledge with thanks and appreciation the stimulus and cooperation received from the Grängesberg Mining Company and the Swedish State Power Board in connection with research reported in Sections 3.1 and 4.1, respectively. I also like to express my appreciation of the collaboration with many colleagues of the Seismological Institute, Uppsala, in preparing the reports listed in the References. As Refs. [8, 9, 20] may not be available to some readers, those interested are invited to request them from the Seismological Institute, Box 517, S-751 20 Uppsala, Sweden.

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